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METHODS OF CALCULATING SOLAR RADIATION VALUES AT BLUE HILL OBSERVATORY, MILTON, MASSACHUSETTS

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ABSTRACT

Methods of calculating direct solar radiation received at normal incidence and the total solar and sky radiation received on a horizontal surface, applicable with accuracies within ± 2.5 percent at Blue Hill Observatory, are presented. As many workers in radiation are content with accuracies within 10 or 15 percent, the same methods might be used with a lesser degree of accuracy in many other areas where atmospheric conditions approximate those at Blue Hill. With data of total solar and sky radiation received on a horizontal surface now available from more than 80 stations in continental United States, Canada, Alaska, and outlying islands, but with few values of normal incidence radiation in the same solar network, it is suggested that these methods can be used to estimate normal incidence for many new regions.

INTRODUCTION

The primary purpose of this paper is to show methods of using the numerous available values of total solar and sky radiation received on a horizontal surface to calculate the amount of solar radiation received at normal incidence. A method also is shown whereby normal incidence data may be used to calculate the total solar radiation received on a horizontal surface; calculated values of diffuse radiation on a horizontal surface may be obtained as a by-product. These methods are developed from radiation records at Blue Hill Observatory for January 1950 through April 1952. A brief review of instrumental equipment furnishing radiation records is made before developing the methods to give the reader an indication of the type and accuracy of available data.

INSTRUMENTAL EQUIPMENT

Records of total solar and sky radiation received on a horizontal surface at numerous stations in the United States, Canada, Alaska, and outlying islands are furnished by the e. m. f. generated by Eppley 180° pyrheliometers [1, 2] and suitable potentiometers. At all the normal inci-

dence stations in this network, with the exception of Table Mountain, Calif., values are obtained by means of Eppley normal incidence pyrheliometers [3, 4] and suitable recording potentiometers. At Table Mountain, the Smithsonian Institution uses silver-disk pyrheliometers to measure the solar constant, while at Blue Hill this type of instrument is used for standardization purposes. Although recording instruments suffice for most climatological purposes, this method of obtaining normal incidence values with a secondary instrument lacks sufficient accuracy to warrant its use in this study. It was necessary to use the recordings of total solar and sky radiation obtained instrumentally, but values of normal incidence radiation measured with the eye-read silver-disk pyrheliometer at Blue Hill were used exclusively in these calculations. Experienced observers can and do read these instruments with an accuracy within one-fourth of one percent [5]. Obvious errors occur in recorded values unless the instrumental equipment is maintained in almost perfect condition, that is, the pyrheliometer compared frequently with a standard and the potentiometer calibrated so that it reads correctly at all points. At Blue Hill, the 180° pyrheliometer is compared periodically with a standard and the potentiometer is checked frequently. That this apparatus is in good condition seems apparent from the results presented in this

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paper. Diffuse radiation on a horizontal surface is measured by means of an occulting ring which continuously shades the diffuse pyrheliometer from the direct rays of the sun, but permits the diffuse or sky radiation to impinge upon the receiving surface at all times [6]. Some of the instrumental sources of errors are listed in the terminal section of the present study.

DEFINITIONS OF SYMBOLS

The following symbols will be used:

I_{hm} is the measured total solar and sky radiation received on a horizontal surface, expressed in langley's/minute [1, 2]. In order to avoid such a cumbersome expression, hereafter it will be termed "total horizontal radiation".

I_{nm} is the measured direct normal incidence radiation, expressed in langley's/minute [3, 4].

D_{hm} is the measured diffuse radiation on a horizontal surface, expressed in langley's/minute [6].

Z is the solar zenith distance expressed in degrees.

I_{nc} is the calculated direct solar radiation received on a surface normal to the sun and expressed in langley's/minute.

I_{hc} is the calculated total solar and diffuse radiation received on a horizontal surface expressed in langley's/minute.

D_{hc} is the calculated diffuse radiation on a horizontal surface expressed in langley's/minute.

CALCULATION METHODS

The calculation methods are to be developed through use of the ratio

$$F = \frac{I_{nm} \cos Z}{I_{hm}} \quad (1)$$

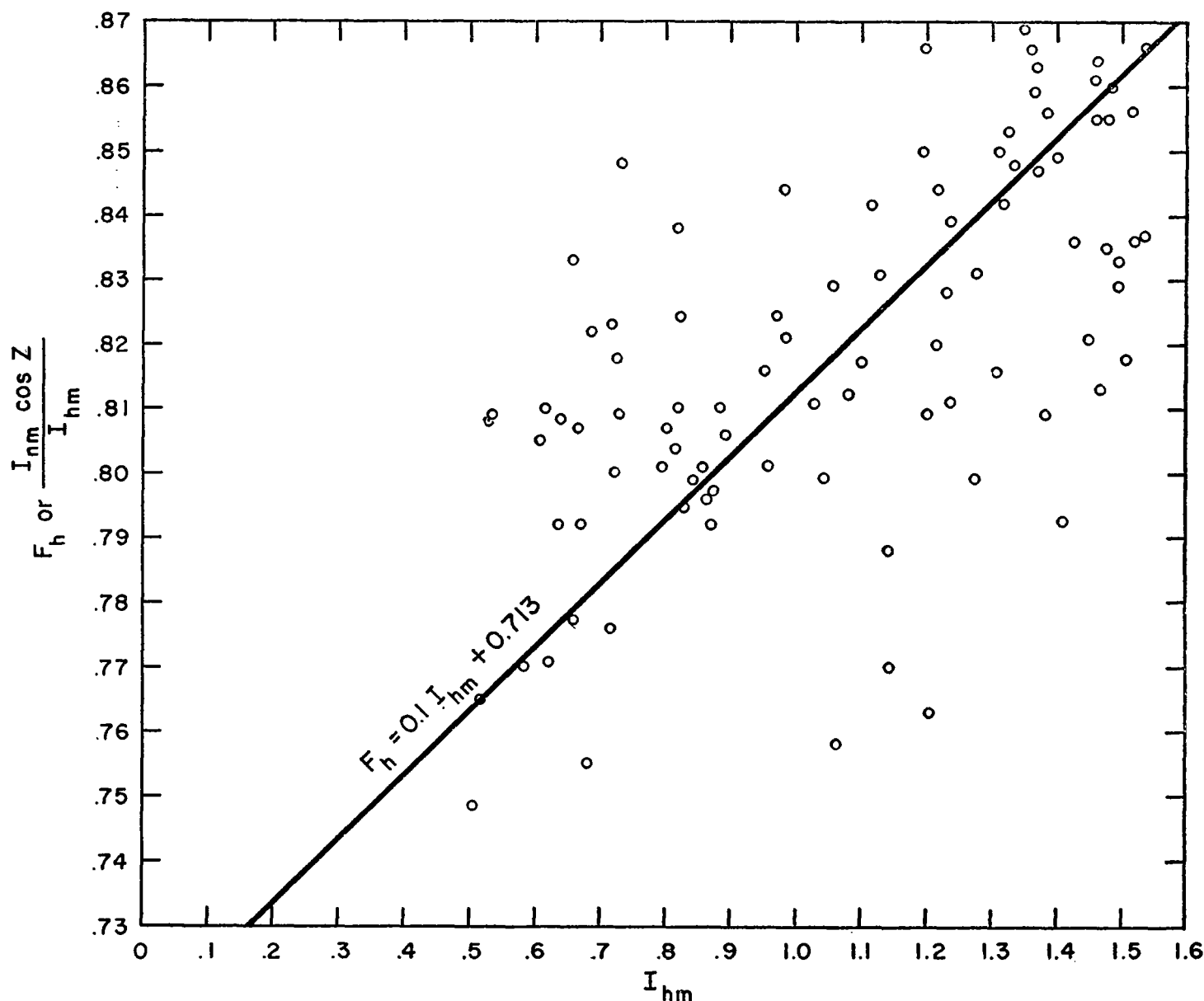


FIGURE 1.—Plot of the ratio $(I_{nm} \cos Z)/I_{hm}$ against I_{hm} . The straight line, fitted by eye, gives equation (4).

Kimball [7] has evaluated this ratio as a function of solar altitude, but for the present purposes we shall first designate it as F_h and empirically evaluate it as a linear function of I_{hm} to be used in computing I_{nc} . Next, we shall designate the ratio as F_n and evaluate it as a linear function of I_{nm} to be used in computing I_{hc} . Thus, the factors F_h and F_n are the critical quantities for the calculation methods. In terms of F_h and F_n , the calculated values I_{nc} and I_{hc} are given respectively by

$$I_{nc} = I_{hm} F_h / \cos Z \quad (2)$$

and

$$I_{hc} = (I_{nm} \cos Z) / F_n \quad (3)$$

To obtain F_h for use in (2) we plot the ratio $(I_{nm} \cos Z) / I_{hm}$ against the measured total horizontal radiation I_{hm} (see fig. 1). Drawing a line of best fit as determined visually we obtain a linear equation,

$$F_h = 0.1 I_{hm} + 0.713 \quad (4)$$

having substituted F_h for $(I_{nm} \cos Z) / I_{hm}$ in accordance with (1).

In a similar manner we obtain F_n for use in (3) by plotting the same ratio against measured normal incidence radiation I_{nm} (fig. 2). Drawing a line of best fit as determined by eye, we find ¹

$$F_n = 0.275 I_{nm} + 0.4475 \quad (5)$$

Finally, by definition of terms, D_{hc} may be calculated from

$$D_{hc} = I_{hm} - I_{nm} \cos Z \quad (6)$$

RESULTS

The Blue Hill data from which figures 1 and 2 and equations (4) and (5) were derived are presented in table 1. The results of computations of I_{nc} , I_{hc} , and D_{hc} are also given in table 1 and are compared with the measured values I_{nm} , I_{hm} , and D_{hm} , respectively. As an example to illustrate the methods described in the preceding section, the computations for January 17, 1950 are given step by step at the foot of table 1.

While the percentage differences between the measured and calculated diffuse radiation are rather large, the average difference in langley's/minute is relatively small, being of the order of 0.07 ly./min. In every case, the calculated value is larger than the measured.

The results for I_{nc} and I_{hc} show promise as evidenced by the small probable errors of the values of calculated from

measured which average less than ± 2.5 percent based on the formula

$$p.e. = 0.6745 \sqrt{\frac{v^2}{n-1}}$$

Obviously the chief cause of the close relationship between the ratios of the two types of radiation values is the secant-effect. Nevertheless, when used with caution we believe this method should prove useful for rough determinations. However, discretion should be exercised in applying these methods to other areas for unless at-

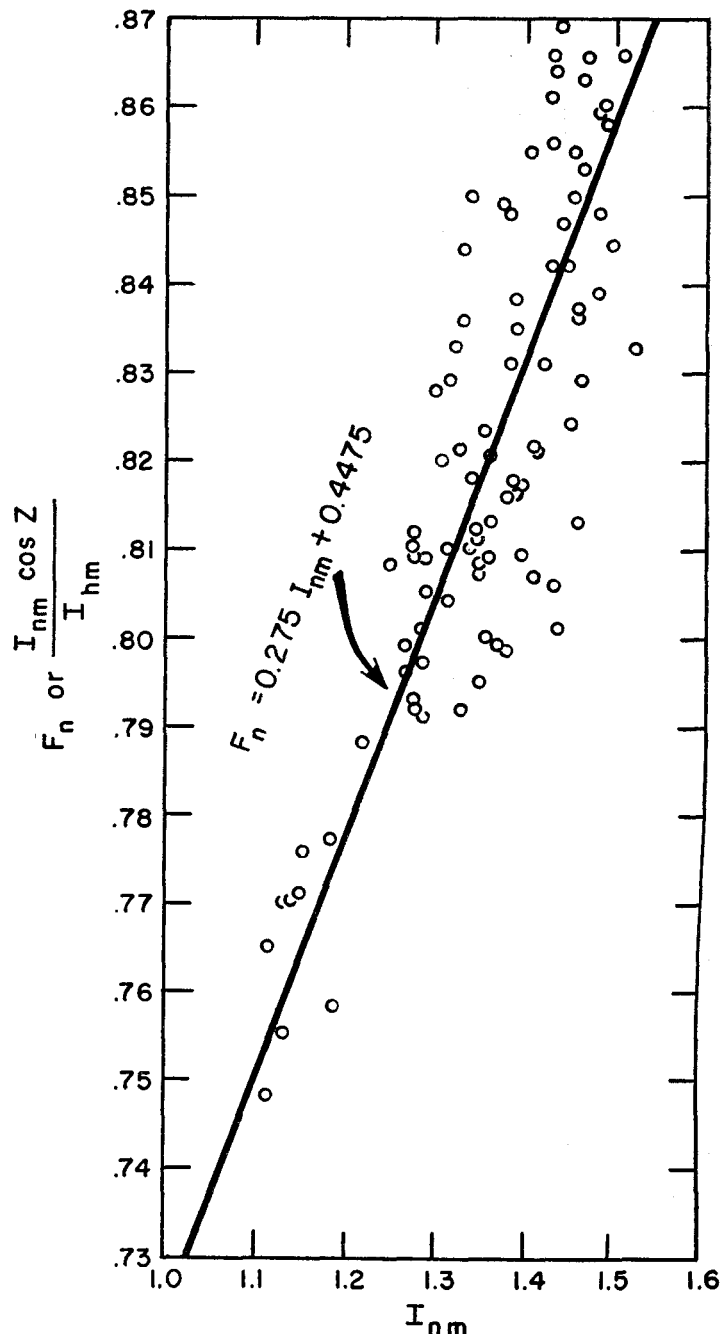


FIGURE 2.—Plot of the ratio $(I_{nm} \cos Z) / I_{hm}$ against I_{nm} . The straight line, fitted by eye, gives equation (5).

¹ EDITORIAL NOTE: It is apparent that equations (4) and (5), with F_h and F_n replaced by $(I_{nm} \cos Z) / I_{hm}$, are not strictly compatible. This inconsistency may be avoided by constructing a single scatter diagram on which values of I_{hm} are plotted against the coordinate values I_{nm} and $\cos Z$. Equally-spaced straight isopleths can then be fitted to the field of I_{hm} , and the resulting chart used to determine either I_{hc} or I_{nc} with an accuracy on the Blue Hill developmental data comparable to that obtained by Hand.

atmospheric conditions approximate those at Blue Hill, serious errors will result. In particular, no attempt should be made to use these methods over large industrial areas as the diffuse radiation then is too large a percentage of the total radiation [8].

SOURCES OF ERROR

In the interpretation of the results for Blue Hill or in the application of the methods to data from other stations, the following sources of instrumental and observational errors should be kept in mind in addition to errors resulting from changes in atmospheric conditions that the methods do not take into account:

- (1) The occulting ring cuts off approximately 5 percent of the sky radiation in addition to shading the instrument from the direct rays of the sun [6].
- (2) The area adjacent to the sun is the brightest portion of the sky [9].
- (3) Errors of Eppley pyrhemometers, while not of great magnitude, include internal reflections from the inner portion of the hemispherical bulb, caustics and striae caused by the glass cover, and variations in efficiency of the thermopile with fluctuations in temperature [10]. And obviously the shaded diffuse pyrhemometer remains cooler than a freely exposed instrument when the sun is shining.

TABLE 1.—Observed and computed solar radiation values at Blue Hill Observatory, Milton, Mass.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Date	Z (deg.)	I_{obs} (ly/min)	I_{obs} (ly/min)	$I_{\text{obs}} \cos Z$ (ly/min)	$I_{\text{obs}} \cos Z$ (ly/min)	F_{obs} (Eq. 4)	I_{obs} (Eq. 2)	$I_{\text{obs}} - I_{\text{obs}}$ (ly/min)	$I_{\text{obs}} - I_{\text{obs}} \times 100\%$ (ly/min)	D_{obs} (Eq. 6)	D_{obs} (ly/min)	F_{obs} (Eq. 5)	I_{obs} (Eq. 3)	$I_{\text{obs}} - I_{\text{obs}}$ (ly/min)	$I_{\text{obs}} - I_{\text{obs}} \times 100\%$ (ly/min)
1950															
Jan. 17	70.1	1.249	0.526	0.425	0.808	0.7656	1.183	-0.066	-5	0.101	0.064	0.7910	0.537	+0.011	+2
Jan. 26	61.1	1.150	.716	.556	.776	.7846	1.162	+0.012	+1	.160	.108	.7638	.728	+0.012	+2
Feb. 3	61.4	1.432	.856	.685	.800	.7986	1.428	-0.004	0	.171	.112	.8413	.815	+0.041	+5
Feb. 3	64.6	1.381	.724	.592	.818	.7854	1.326	-0.055	-4	.132	.104	.8273	.716	+0.008	+1
Feb. 3	73.0	1.272	.458	.372	.812	.7588	1.189	-0.083	-7	.086	.064	.7973	.466	+0.008	+2
Feb. 20	60.6	1.368	.840	.672	.800	.7970	1.364	-0.004	0	.168	.145	.8237	.815	+0.025	+3
Mar. 6	67.8	1.324	.632	.500	.791	.7762	1.298	-0.027	-2	.132	.112	.8116	.616	+0.016	+3
Mar. 19	60.7	1.345	.828	.658	.795	.7958	1.346	+0.001	0	.170	.107	.8174	.805	+0.023	+3
Apr. 10	34.8	1.451	1.466	1.191	.812	.8596	1.535	+0.084	+6	.275	.210	.8465	1.408	+0.058	+4
Apr. 10	35.1	1.518	1.491	1.242	.833	.8621	1.571	+0.053	+3	.249	.210	.8650	1.436	+0.055	+4
Apr. 27	28.6	1.272	1.408	1.117	.793	.8538	1.369	+0.097	+8	.291	.200	.7973	1.401	+0.007	0
May 8	40.0	1.454	1.310	1.114	.850	.8440	1.443	-0.012	-1	.196	.085	.8474	1.315	+0.005	0
May 8	32.4	1.443	1.460	1.252	.858	.8590	1.485	+0.002	0	.208	.104	.8553	1.464	+0.004	0
May 8	29.4	1.442	1.514	1.265	.836	.8644	1.502	+0.050	+3	.249	.115	.8468	1.494	+0.020	+1
May 8	42.3	1.374	1.272	1.016	.799	.8402	1.445	+0.071	+5	.256	.116	.8254	1.231	+0.041	+3
May 9	36.3	1.437	1.368	1.158	.846	.8498	1.442	+0.005	0	.210	.104	.8427	1.374	+0.006	0
May 9	39.8	1.385	1.304	1.064	.816	.8434	1.432	+0.047	+3	.240	.123	.8284	1.284	+0.020	+2
May 14	27.6	1.387	1.472	1.229	.835	.8602	1.429	+0.042	+3	.243	.115	.8289	1.483	+0.011	+1
May 17	23.2	1.337	1.502	1.229	.818	.8632	1.411	+0.074	+6	.273	.178	.8152	1.507	+0.005	0
May 22	27.9	1.451	1.532	1.282	.837	.8662	1.502	+0.051	+4	.250	.098	.8465	1.515	+0.017	+1
May 22	24.7	1.424	1.512	1.294	.856	.8642	1.438	+0.014	+1	.218	.100	.8391	1.542	+0.030	+2
May 22	52.3	1.321	.984	.808	.821	.8114	1.306	-0.015	-1	.176	.105	.8108	.996	+0.012	+1
June 18	28.0	1.427	1.458	1.260	.864	.8588	1.418	-0.009	-1	.198	.100	.8399	1.500	+0.042	+3
June 18	19.8	1.434	1.552	1.349	.869	.8682	1.432	-0.002	0	.203	.113	.8419	1.603	+0.051	+3
July 27	37.0	1.150	1.204	.918	.762	.8334	1.256	+0.106	+9	.286	.232	.7638	1.202	+0.002	0
Aug. 8	40.2	1.301	1.212	.994	.820	.8342	1.324	+0.023	+2	.218	.118	.8052	1.234	+0.022	+2
Aug. 8	25.6	1.324	1.428	1.194	.836	.8558	1.355	+0.031	+2	.234	.116	.8116	1.471	+0.043	+3
Sept. 7	48.8	1.262	1.040	.831	.799	.8170	1.290	+0.028	+2	.209	.116	.7946	1.046	+0.006	0
Sept. 7	38.4	1.299	1.230	1.018	.828	.8360	1.312	+0.013	+1	.212	.114	.8047	1.265	+0.035	+3
Sept. 17	39.7	1.446	1.320	1.112	.842	.8450	1.450	+0.005	0	.208	.075	.8449	1.316	+0.004	0
Sept. 17	41.7	1.416	1.272	1.057	.831	.8402	1.431	+0.015	+1	.215	.087	.8369	1.263	+0.009	+1
Sept. 17	51.7	1.343	1.026	.832	.811	.8156	1.350	+0.007	+1	.194	.070	.8168	1.019	+0.007	+1
Oct. 5	49.7	1.358	1.080	.878	.813	.8210	1.371	+0.013	+1	.202	.127	.8210	1.070	+0.010	+1
Oct. 5	53.7	1.288	.952	.763	.801	.8082	1.300	+0.012	+1	.189	.138	.8017	.951	+0.001	0
Oct. 31	57.3	1.284	.870	.694	.798	.8000	1.288	+0.004	0	.176	M	.8006	.866	+0.004	0
Nov. 6	60.3	1.283	.794	.636	.801	.7924	1.270	-0.013	-1	.158	.102	.8003	.794	0	0
Nov. 6	65.4	1.148	.620	.478	.771	.7750	1.164	+0.009	+1	.142	.102	.7632	.626	+0.006	+1
Nov. 13	63.1	1.135	.680	.514	.756	.7810	1.174	+0.039	+3	.166	M	.7596	.676	+0.004	+1
Nov. 13	69.2	1.112	.516	.395	.766	.7646	1.111	-0.001	0	.121	M	.7533	.524	+0.008	+2
1951															
Jan. 9	65.6	1.284	.670	.530	.791	.7800	1.265	-0.019	-1	.140	.120	.8006	.663	+0.007	+1
Jan. 9	71.6	1.142	.468	.360	.769	.7598	1.127	-0.015	-1	.108	.098	.7616	.473	+0.005	+1
Feb. 9	62.7	1.408	.800	.646	.807	.7930	1.383	-0.025	-2	.154	.101	.8347	.774	+0.026	+3
Feb. 9	56.4	1.494	.980	.827	.844	.8110	1.436	-0.058	-4	.153	.106	.8584	.963	+0.016	+2
Feb. 16	60.2	1.334	.818	.663	.810	.7948	1.308	-0.026	-2	.155	.113	.8144	.814	+0.004	0
Mar. 27	40.0	1.475	1.333	1.130	.848	.8463	1.473	-0.002	0	.203	.105	.8531	1.324	+0.009	+1
Mar. 27	49.4	1.393	1.110	.907	.817	.8240	1.405	+0.012	+1	.203	.107	.8306	1.092	+0.018	+2
Apr. 23	31.9	1.455	1.490	1.235	.829	.8620	1.512	+0.057	+4	.255	.162	.8476	1.457	+0.033	+3
Apr. 30	36.6	1.462	1.356	1.174	.866	.8486	1.433	-0.029	-2	.182	.120	.8496	1.382	+0.026	+2
Apr. 30	35.6	1.452	1.368	1.181	.863	.8498	1.430	-0.022	-2	.187	.110	.8468	1.394	+0.026	+2
May 3	41.1	1.376	1.236	1.037	.839	.8366	1.454	+0.078	+6	.199	.118	.8259	1.255	+0.019	+2
May 3	28.2	1.421	1.454	1.252	.861	.8584	1.416	-0.005	0	.202	.130	.8383	1.494	+0.040	+3
May 9	40.5	1.332	1.192	1.013	.850	.8322	1.305	-0.027	-2	.179	.133	.8138	1.245	+0.053	+4
May 13	57.2	1.266	.862	.686	.796	.7992	1.272	+0.006	0	.176	.111	.7956	.862	0	0
May 14	39.1	1.324	1.218	1.027	.843	.8348	1.310	-0.014	-1	.191	.143	.8116	1.266	+0.048	+4
May 14	25.8	1.400	1.475	1.260	.854	.8605	1.410	+0.010	+1	.215	.152	.8325	1.514	+0.039	+3
May 14	30.1	1.373	1.399	1.188	.849	.8529	1.379	+0.006	0	.211	.152	.8251	1.440	+0.041	+3
Aug. 2	38.8	1.421	1.315	1.107	.842	.8445	1.425	+0.004	0	.208	M	.8383	1.321	+0.006	0
Sept. 25	55.8	1.270	.881	.714	.810	.8011	1.256	-0.014	-1	.167	M	.7968	.896	+0.015	+2
Sept. 26	55.9	1.385	.951	.776	.816	.8081	1.371	-0.014	-1	.175	.128	.8284	.937	+0.014	+2
Nov. 4	57.2	1.272	.870	.689	.792	.8000	1.285	+0.013	+1	.181	.112	.7973	.864	+0.006	+1
Nov. 9	65.5	1.316	.655	.546	.833	.7785	1.230	-0.086	-7	.109	.085	.8094	.674	+0.019	+2
Nov. 9	60.2	1.377	.817	.684	.837	.7947	1.307	-0.071	-5	.133	.087	.8262	.828	+0.011	+1
Nov. 25	64.2	1.350	.714	.588	.823	.7844	1.287	-0.063	-5	.126	.081	.8188	.718	+0.004	+1
Nov. 27	63.1	1.372	.732	.621	.848	.7862	1.272	-0.100	-7	.111	.085	.8248	.753	+0.021	+3
Dec. 4	67.4	1.345	.640	.517	.808	.7770	1.297	-0.048	-4	.123	.080	.8174	.632	+0.008	+1
Dec. 4	64.3	1.183	.660	.513	.777	.7790	1.186	+0.003	0	.147	.102	.7728	.664	+0.004	+1
Dec. 11	67.6	1.285	.608	.490	.806	.7738	1.235	-0.050	-4	.118	.077	.8009	.611	+0.003	0
Dec. 27	65.4	1.358	.688	.565	.821	.7818	1.292	-0.066	-5	.123	.110	.8210	.689	+0.001	0

TABLE 1.—Observed and computed solar radiation values at Blue Hill Observatory, Millon, Mass.—Continued

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Date	Z (deg.)	I_{am} (ly/min)	I_{am} (ly/min)	$I_{\text{am}} \cos Z$ (ly/min)	$I_{\text{am}} \cos Z$ (ly/min)	F_{h} (Eq. 4)	I_{a} (Eq. 2) (ly/min)	$I_{\text{a}} - I_{\text{am}}$ (ly/min)	$\frac{I_{\text{a}} - I_{\text{am}}}{I_{\text{am}}} \times 100\%$	D_{a} (Eq. 6) (ly/min)	D_{a} (ly/min)	F_{h} (Eq. 6)	I_{a} (Eq. 3) (ly/min)	$I_{\text{a}} - I_{\text{am}}$ (ly/min)	$\frac{I_{\text{a}} - I_{\text{am}}}{I_{\text{am}}} \times 100\%$
1952															
Jan. 4.....	70.2	1.112	.504	.377	.748	.7634	1.136	+ .024	+2	.127	.096	.7533	.500	-.004	-1
Jan. 4.....	66.8	1.136	.582	.448	.770	.7712	1.139	+ .003	0	.134	.113	.7599	.589	+ .007	+1
Jan. 11.....	70.2	1.275	.534	.432	.809	.7764	1.208	-.067	-5	.102	.073	.7981	.541	-.007	-1
Jan. 11.....	64.9	1.393	.730	.591	.809	.7860	1.353	-.040	-3	.139	.087	.8306	.711	-.019	-3
Jan. 11.....	64.7	1.352	.722	.578	.800	.7852	1.327	-.025	-2	.144	.083	.8193	.705	-.017	-2
Feb. 12.....	67.6	1.310	.616	.499	.810	.7746	1.252	-.058	-4	.117	.070	.8078	.618	+ .002	0
Feb. 12.....	61.3	1.409	.824	.677	.821	.7954	1.365	-.044	-3	.147	.087	.8350	.810	-.014	-2
Feb. 12.....	56.5	1.449	.970	.800	.824	.8100	1.423	-.026	-2	.170	.091	.8460	.945	-.025	-3
Feb. 12.....	59.7	1.428	.894	.720	.806	.8024	1.422	-.006	0	.174	.091	.8402	.857	-.037	-4
Feb. 12.....	66.4	1.346	.668	.539	.807	.7798	1.301	-.045	-3	.129	.087	.8177	.659	-.009	-1
Feb. 13.....	60.1	1.309	.812	.653	.804	.7942	1.294	-.015	-1	.159	.112	.8075	.808	-.004	0
Apr. 4.....	42.2	1.215	1.142	.900	.788	.8272	1.275	+ .060	+5	.242	.172	.7816	1.152	+ .010	+1
Apr. 16.....	47.1	1.380	1.130	.939	.831	.8260	1.371	-.009	-1	.191	.092	.8270	1.136	+ .006	+1
Apr. 17.....	48.0	1.310	1.058	.877	.829	.8188	1.295	-.015	-1	.181	.179	.8078	1.085	+ .027	+3
Apr. 17.....	41.5	1.340	1.238	1.004	.811	.8368	1.383	+ .043	+3	.234	.124	.8160	1.220	-.008	-1
Apr. 17.....	34.7	1.358	1.380	1.116	.809	.8510	1.428	+ .070	+5	.264	.154	.8210	1.360	-.020	-2
Apr. 18.....	46.9	1.181	1.064	.907	.758	.8194	1.276	+ .095	+8	.257	.121	.7723	1.045	-.019	-2
Apr. 18.....	40.5	1.277	1.200	.971	.809	.8330	1.315	+ .038	+3	.229	.124	.7987	1.216	+ .016	+1
Apr. 24.....	35.9	1.450	1.372	1.175	.856	.8502	1.440	-.010	-1	.197	.122	.8463	1.388	+ .016	+1
Apr. 30.....	43.4	1.429	1.199	1.038	.866	.8329	1.374	-.055	-4	.161	.106	.8405	1.235	+ .036	+3
Apr. 30.....	39.1	1.460	1.328	1.133	.853	.8458	1.447	-.013	-1	.195	.112	.8490	1.335	+ .007	+1
Apr. 30.....	30.8	1.482	1.480	1.273	.860	.8610	1.484	+ .002	0	.207	.125	.8551	1.489	+ .009	+1
Apr. 30.....	27.9	1.501	1.532	1.327	.866	.8662	1.502	+ .001	0	.205	.095	.8603	1.542	+ .010	+1
Apr. 30.....	32.3	1.408	1.449	1.190	.821	.8579	1.471	+ .063	+4	.259	.094	.8347	1.426	-.023	-2
Apr. 30.....	37.7	1.477	1.360	1.169	.859	.8490	1.459	-.018	-1	.191	.090	.8637	1.369	+ .009	+1

Sample computation for Jan. 17, 1950

Given: $Z=70.1^\circ$, $\cos Z=0.34038$ $I_{\text{am}}=1.249$ ly/min $I_{\text{a}}=0.526$ ly/min $D_{\text{a}}=0.084$ ly/min

Step 1: From equation (4),

 $F_{\text{h}}=0.1(0.526)+0.713=.7656$

Step 2: From equation (2),

 $I_{\text{a}}=(0.526)(.7656)/0.34038=1.183$ Step 3: Difference $=I_{\text{a}}-I_{\text{am}}=1.183-1.249=-0.066$ Step 4: Percent difference $=\frac{-0.066}{1.249} \times 100\% = -5\%$

Step 5: From equation (6),

 $D_{\text{a}}=0.526-(1.249)(0.34038)=0.101$

Step 6: From equation (5),

 $F_{\text{h}}=0.275(1.249)+0.4475=0.7910$

Step 7: From equation (3),

 $I_{\text{a}}=(1.249)(0.34038)/0.7910=0.537$ Step 8: Difference $=I_{\text{a}}-I_{\text{am}}=0.537-0.526=0.011$ Step 9: Percent difference $=\frac{0.011}{0.526} \times 100\% = 2\%$

- (4) Errors arising from instrumental recording. In this particular case, the recording pyr heliometer used for diffuse radiation lacks the accuracy of the recorder for measuring total horizontal radiation. Differences in the same direction and of approximately the same magnitude were found by Dr. C. F. Brooks and Miss S. H. Wollaston in an unpublished study.
- (5) The human error ordinarily is an important factor, but in this case we have attempted to minimize this effect by double-checking integration of records and the various calculations.

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